THE BETZ AREA RATIO
A COMPARISON STANDARD FOR WIND-POWER DEVICES

The recent push toward renewable energy and "green" technology has resulted in a wide variety of devices for extracting energy from the wind. The claims of developers are difficult to put into perspective. The following is intended to put forward a performance standard which can be used to rate such devices. It is based upon the upper limit of power attainable from the most commonly used wind-power device...the axial-flow wind turbine.

The time rate at which one collects the total kinetic energy of the wind (that is, by bringing the air to a standstill) passing through a portal with capturing area, A, is

$$P = \Phi A$$

where Φ is the famous "one-half rho v cubed" energy flux which rises with the air density and with the cube of the true wind velocity (not apparent wind). And, real systems only collect a fraction of that power. The analysis called Betz' Law proves that an axial-flow wind turbine is limited to 16/27ths of the above power, referred to here as P_B, flowing through the area swept out by the rotor blades, A_B.

$$P_B = \frac{16}{27} \Phi A_B$$

If a system, such as a kite-power system, generates power from the wind and we set P_B equal to that power, then, in order to generate the same amount of power, a Betz-limited turbine would need a rotor area of

$$A_B = \frac{27}{16} \frac{P_B}{\Phi}$$

It is proposed here, that the system under consideration generating power P_B with a characteristic area, A_S, (such as wing area in a kite-power system) has a Betz area ratio of

$$R_B = \frac{A_B}{A_S}$$

It is the intent of this comparative standard to reduce or eliminate the ambiguity of differences in wing area and wind velocity when comparing the performances of different wind-power systems.

Some interpretation may be required to establish the characteristic area of the system. It would be straight forward to use the wing area if the air vehicle(s) in a kite-power system possessed more or less flat, planar wings which undergo unidirectional motion, such as the moving-root kite system proposed in Project Sea Tree. Analysis of that system indicates Betz area ratios well in excess of 200.

But, if a curved flexible wing is used, one might use the projection of the wing area onto a plane perpendicular to the tether. With the use of multiple tethers, one would use the projection of the wing area onto a plane perpendicular to the sum of the tether tension vectors. In the case of a tethered autogyro, one might use the area swept out by the rotor blades, as would be the case for a wind turbine. But, when the wing sweeps out a circular arc with radius much larger than the wing span, it might make more sense to use the wing area rather than the area of the circle being swept out.

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